Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Students' errors in refractive thinking (the component of identifying problems) about spheres and distance in three-space

To cite this article: I N Yenti *et al* 2020 *J. Phys.: Conf. Ser.* **1657** 012061

Students' errors in refractive thinking (the component of identifying problems) about spheres and distance in three-space

I N Yenti[1,2], Y S Kusumah[1], J A Dahlan[1] and H Fitri[3]

[1]Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung, Indonesia
[2]Institut Agama Islam Negeri Batusangkar, Jl. Sudirman No.137, Batusangkar, Indonesia
[3]Institut Agama Islam Negeri Bukittinggi, Jl. Gurun Aur Kubang Putih, Indonesia

E-mail: isranurmaiyenti@iainbatusangkar.ac.id

Abstract. Identifying a problem is the fundamental component in the refractive thinking process. This activity is carried out after students read the given mathematical problem. Failure of students to identify problems will affect problem-solving strategies. Some strategies in solving problems can be found by identifying the information in the issues correctly. This study aims to reveal the errors of students in refractive thinking on the component identifying problems. The research subjects were 43 students who took the Multivariable Calculus course at one of the Islamic Higher Education Institutions in West Sumatra. The process of collecting data uses tests and questionnaires. This study found several errors in refractive thinking made by students when identifying problems, namely reading errors, language interpretation errors, comprehension errors, factual errors, and errors in visualizing mathematical graphics.

1. Introduction

Refractive thinking is one of the higher order thinking abilities, because refractive thinking occurs through a critical thinking process. Refractive thinking is a thought process that produces decisions through reflective thinking and critical thinking [1,2]. The refractive thinking process is illustrated in Figure 1.

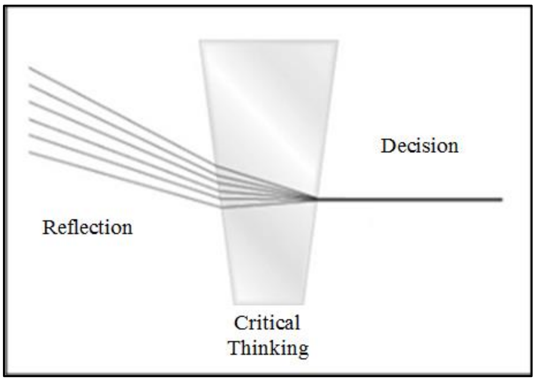


Figure 1. The refractive thinking process (adopted from [2, 3])

Figure 1 shows the refractive thinking process that occurs because the process of reflective thinking, critical thinking so as to produce decisions. The refractive thinking process begins with reflective thinking. When students solve a problem, they find several ways to solve problems. This is seen in the lines that appear in the reflection section (as reflective thinking) which symbolize the existence of several ways to solve the problem. After that, students analyze several strategies for solving the problems, which indicate the occurrence of critical thinking processes. These strategies are combined with other strategies to find a decision.

Refractive thinking consists of 3 components, namely identifying problems, formulating strategies and conducting an evaluation [4]. Identifying a problem means listing the information provided in the problem, grouping the information, visualizing the information in mathematical symbols and identifying some of the concepts or principles contained in the problem. Formulating a strategy can be done by identifying the relationship between information, several statements or concepts, and elements that are considered important in solving problems, even proposing some possible solutions to problems. Conducting an evaluation means assessing the information generated based on relevant information to make conclusions.

Identifying a problem is the initial component in the refractive thinking process. Identifying a problem is carried out after students read the given mathematical problem [4]. There are 3 indicators used in identifying problems, namely: rewriting information on a given problem (indicator 1), interpreting mathematical situations rationally (indicator 2), and representing ideas in the form of mathematical symbols or pictures (indicator 3).

However, a study reveals that most students who study the Application of Integral concepts experience errors at the stage of understanding the problem because they do not read and understand questions carefully [5]. Meanwhile, errors that students can make when understanding a problem are factual errors, errors due to habits and language interpretation [6]. Thus, the two studies above found that students have errors when understanding a problem.

Based on the results of research by Solfitri and Maimunah [5] and Widodo [6], it is very important to collect data on the errors the students have in refractive thinking to identify problems. Failure of students to identify problems will affect problem solving strategies. If students can understand the problem well, they can solve the problem [7]. Some strategies in solving problems can be found by identifying the information in the problems correctly.

Research on student errors can help lecturers in developing pedagogical techniques that can overcome student difficulties in their learning [8, 9] and as an evaluation tool in the learning process so that learning objectives can be optimally achieved [10]. This research is focused on discussing students' errors in refractive thinking, particularly the component of identifying problems for spheres and distance in three-space. The results of this study are expected to provide an overview in the selection of learning approaches, in order to develop students' mathematical refractive thinking skills.

2. Method

This research is a descriptive study with 43 research subjects taking Multivariable Calculus courses at one of the Islamic Higher Education Institutions in West Sumatra. Data were collected by using tests and questionnaires. Students were given a complex problem in tests of mathematical refractive thinking abilities on spheres and distances in three-space. The problem is "A sphere with a center point $(4, 2, 6)$ is located in the first octane. The sphere tangent to one of the coordinate-plane. If the point P is the tangent point of the sphere in the coordinate-plane, determine the distance of the point P to the origin." Students were asked to answer the question, "Is the information from the distance problem above sufficient? If so, identify and interpret the given mathematical situation." To find out the student's mathematical refractive thinking disposition when identifying a problem, they were asked to fill in a questionnaire about: 1) Their difficulties when searching for information from the given distance problem; 2) Did they find the required information from the given distance problem so that you were able to determine the strategy for solving the problem.

3. Result and Discussion

The students' answers to the problem of refractive thinking were given a score of 0, 1, and 2. A score 0 means the answer is incorrect. A score 1 means the answer is partially correct. A score 2 means all the answers are correct. The percentage score for each indicator on identifying problems in refractive thinking is presented in Table 1.

Table 1. Percentage of Student Scores when Identifying Problems in Refractive Thinking

Indicator	Student scores		
	0	1	2
1	4.7	88.4	7.0
2	51.2	48.8	0.0
3	9.3	83.7	7.0

Table 1 shows about 93% of students who have not rewritten the information given in full, no one is able to interpret mathematical situations rationally correctly, and about 7% of students are able to represent ideas in the form of mathematical symbols or drawings. In general, the students ignore the information "the sphere tangent to one of the coordinate-plane". In fact, this information is very useful in finding the radius of the sphere or the coordinates of the tangent point *P*.

Further, tracing the students' activities in identifying problems is done through a refractive thinking disposition questionnaire. Almost all students admitted that they were able to find the required information for solving the problem. They did read questions and making pictures repeatedly. However, when it was verified by the answer key, most of them ignore the most important information. This causes them having difficulty in solving problems.

3.1. Error in rewriting information on a given problem

There were 2 errors made by students in rewriting information on a given problem. First, the error in writing the information obtained from the problem was 84%. Secondly, there was no error in rewriting the questions asked by the questions, because 84% of students did not rewrite the questions asked.

In general, mistakes made by students in rewriting things that are known are students not writing information about the sphere tangent to one of the coordinate-plane. Almost all students only write information about the center of the sphere and the tangent point *P*, not the tangent point *P* in the coordinate-plane. Whereas 14% of students did not write down what was known and only 2% of students wrote it correctly. The following are excerpts from students' answers in rewriting things that are known and things that are asked.

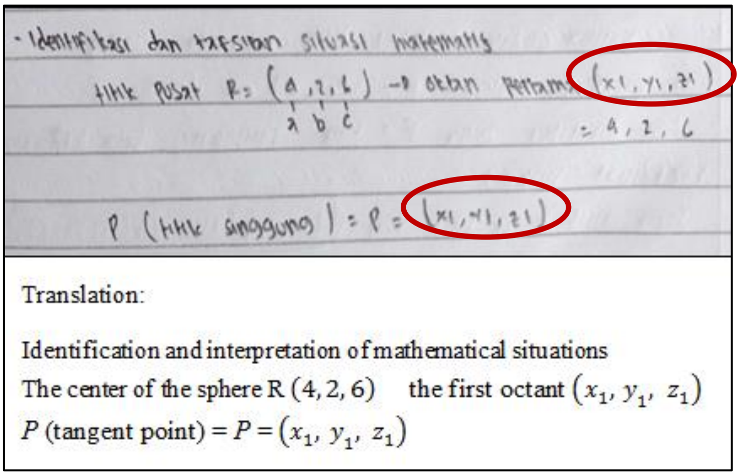
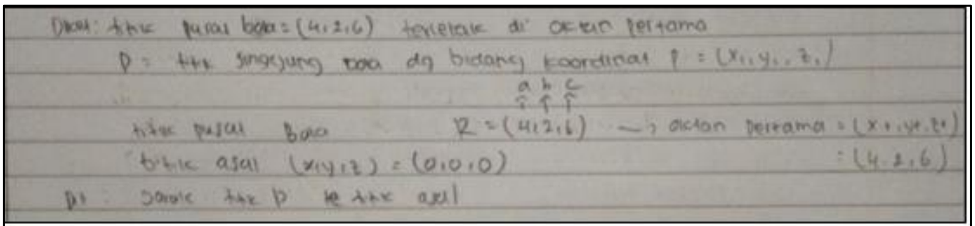


Figure 2. Example of Wrong Answer in Rewriting Problem Information

In Figure 2, the students only wrote down everything that can be identified. The answer is still incomplete, however, due to the absence of information concerning sphere tangent to one of the coordinate planes and equating the coordinates of the tangent point P with the center point of the sphere, namely (x_1, y_1, z_1). Figure 3 shows a student's correct answer in writing things that are known and things that are asked.



Translation:
Known: The center of the sphere = $(4, 2, 6)$ in the first octant
P = tangent point of the sphere in the coordinate-plane
The center of the sphere $R\ (4, 2, 6)$ the first octant
The original point $(x, y, z) = (0, 0, 0)$
Asked: The distance between the point P and the original point

Figure 3. Example of Correct Answer in Rewriting Problem Information

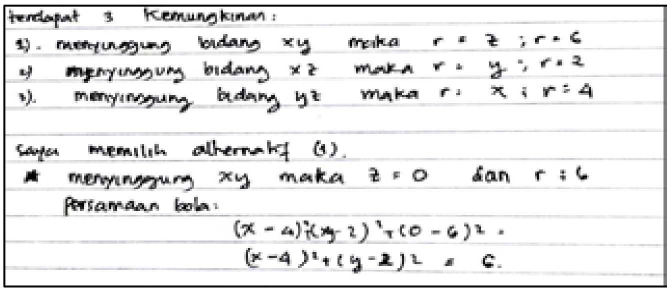
The students' errors in rewriting information could be caused by their carelessness in reading questions [5] and their carelessness in using the information given [11]. Reading errors occur when students are unable to read the given problem and identify the mathematical sentences and symbols contained in the problem [12, 13].

3.2. Error in interpreting mathematical situations rationally

Students' error in interpreting mathematical situations rationally, including: 91% in interpreting the language of the problems, 5% in interpreting information on the sphere tangent to one of the coordinate-plane, 79% in interpreting point P are tangent point in the coordinate-plane, 9% in interpreting the center of the sphere, 23% errors in interpreting the origin and 86% errors in understanding the distance. The highest percentage that causes errors in interpreting mathematical situations rationally is an error in interpreting the language of the problem.

Language interpretation error is an error the students have in interpreting problems, including changing mathematical problems in the form of everyday language into mathematical sentences [6]. In this study, most students only focus on the center of the sphere and the tangent point P, but ignore the condition of the sphere that tangent to one of the coordinate-plane. They stated that the information contained in the problem was still incomplete because the radius of the sphere and the coordinates of the point P were unknown.

Student errors in interpreting information about the sphere tangent to one of the coordinate-plane is still very little. This is caused by 95% of students, who did not write the information provided so they did not interpret the information. There are 2 students who are correct in interpreting information about the sphere tangent to one of the coordinate-plane. One student explains the reason for choosing a tangent point P, while another student has no explanation. The students' answers are in Figure 4.



Translation:

There are 3 ways:
1. The tangent to the xy-plane, then $r = z; r = 6$
2. The tangent to the xz-plane, then $r = y; r = 2$
3. The tangent to the yz-plane, then $r = x; r = 4$

I choose the first way
The tangent to the xy-plane, then $z = 0$ and $r = 6$
The equation of a sphere

Figure 4. Correct Answers in Meaning the Sphere Tangent to One of the Coordinate-Plane

Figure 4 shows how the students interpreted the sphere tangent to one of the coordinate-plane in 3 ways. One of the students decided to choose the first way, that is the tangent to the *xy*-plane. This choice is wrong, because when the sphere has a radius of 6, then the sphere is not only a tangent to the coordinate-plane but also intersects the *xz*-plane and *yz*-plane. While, it is known from the problem, that the sphere is only the tangent to one of the coordinate-plane.

The cause of student errors in interpreting point *P* as tangent point in the coordinate-plane is that the students only assume the point *P* in general, which can be seen from the example coordinates of the point *P* used. If the point *P* is the tangent point in the coordinate-plane, the coordinates of the point *P* containing one of them are 0. Examples of student answers that are wrong in assuming the coordinates of the point *P* can be seen in Figure 5.

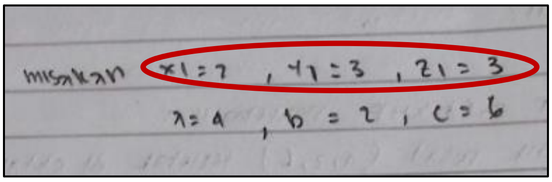


Figure 5. Wrong Student Answers in Assuming the Coordinates of the Tangent Point *P*

Figure 5 is a continuation of students' answers in Figure 2. They assume the coordinates of the point *P* is $(2, 3, 3)$. Thus, the point *P* $(2, 3, 3)$ does not tangent to the coordinate-plane.

The student errors in interpreting the origin are still few, because 67% of students do not provide answers and interpretations of the origin. Student errors in interpreting the origin can be seen from their answers which equate the coordinates of the origin with the center of the sphere, but the values z_1 and z_2 are exchanged (see Figure 6).

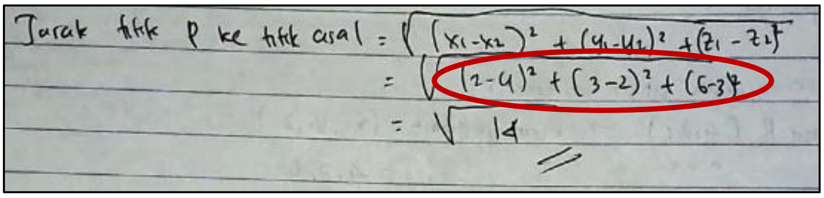
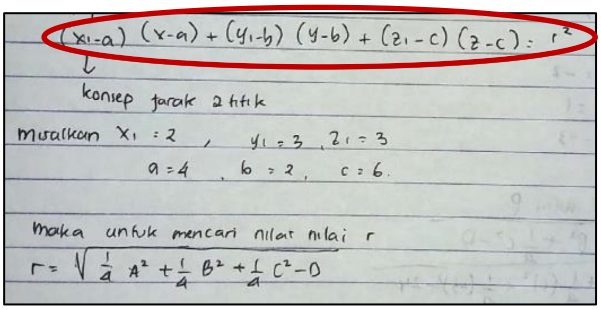
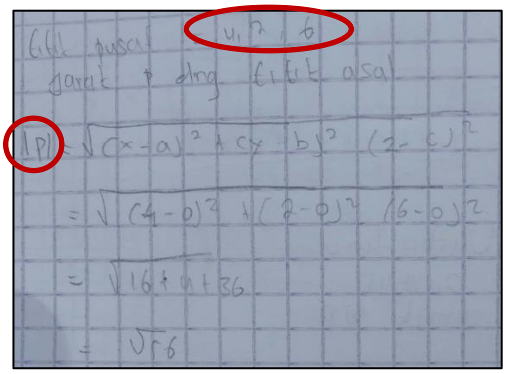


Figure 6. Error in Using Origin Points Coordinates

The final error found in interpreting mathematical situations rationally is the error of understanding distance. The errors made by the students in interpreting distance include: 1) using the concept of distance that resembles a sphere equation; 2) using absolute values to express the difference of each coordinate, and 3) incorrectly using distance symbol.



a



b

Figure 7. Error in Meaning Distance

In general, the students interpret the distance as shown in Figure 7a. The equation resembles a sphere equation, but it is not a correct sphere equation concept. This can occur because they are not able to distinguish between relevant and irrelevant information or unable to gather the required information [11]. Meanwhile, the distance formula used by a student in Figure 7b is correct. However, the way he used the distance symbol is still incorrect.

Based on the explanation of the results above, it appears that the students make errors in interpreting mathematical situations in a rational way: the meaning of information and the information that should be revealed. This error can occur because they misunderstand certain instructions or keywords, or they have difficulty in using the correct information [11]. These kinds of errors can also be called as comprehension errors. Comprehension errors are students' errors related to symbols, expressions, and problems given in questions [14].

3.3. Error in representing ideas in the form of mathematical symbols or images

The students' errors in representing ideas in the form of mathematical symbols or drawings, consisting of: 14% errors in using symbol from the center of the sphere, 7% errors in using the origin point symbol, 12% errors in expressing point coordinates and 9% errors in making a spherical graph. The least errors made by students in presenting ideas are errors using the origin point symbol. This is caused by 88% of students not writing the origin symbol. Likewise, for errors in making a spherical graph, only 6 people presented it in graphic form.

The errors made by the students in symbolizing the center of the sphere include: 1) using the letter P, whereas the letter P based on the problem above is the tangent point; and 2) using the letter r, while r is a symbol of the radius of the sphere. The students' error in stating the point coordinates is the use of curly brackets and not using brackets (can be seen in Figure 7b).

The students' errors in representing ideas in the form of symbols can be caused by factual errors [10]. Brown and Skown in [10,15] stated that factual error is an error that occurs when the student doesn't understand the information in the question such as terms and symbols. In addition, errors in representing ideas in the form of symbols can occur due to errors in reading. Reading errors occur when written words or symbols fail to be recognized by students which causes failure to solve problems [7, 16].

The students' errors in representing ideas in the form of images can be caused by lack of visualization skills and basic geometrical knowledge. The ability of visualization and knowledge of basic geometric diagrams are important in the solution process of many mathematical problems (especially word problems) in applied calculus [17]. The prerequisites of visual thinking in calculus (applied calculus) include the ability to extract specific information from diagrams, algebraic understanding (variables, equations, formulas) and geometry (planes and spaces) as alternative languages for the expression of mathematical ideas, and knowledge of rules and conventions which is related to mathematical graphs [18]. The research finding [17] is a failure to visualize geometric diagrams of word problems tends to prevent students from getting the required formula.

The cause of the students’ errors in refractive thinking on the component of identifying problems is traced through a questionnaire of refractive thinking disposition that has been filled out. The difficulties experienced by the students when they searched for information from a given problem are language problems that are difficult to understand, incomplete information provided about problems (for example: the coordinates of the tangent point P are unknown), do not understand and have forgotten material about the sphere and the distance in in three-space, and working for the first time on the problem of refractive thinking. This is consistent with the opinion [19], which states that there are two reasons students can make errors, namely 1) lack of knowledge which has an effect on procedural, factual, and conceptual errors and 2) lack of attention and carelessness.

Thus, through tests and questionnaires show the same results, each of which is a complement one to another. The mistakes made by the students in refractive thinking on the components of identifying problems are reading errors, language interpretation errors, comprehension errors, factual errors and errors in visualizing geometric graphs. Reading errors and comprehension errors are two errors in Newman Error Analysis (NEA) [5, 7, 11-14, 16]. NEA is a model used to analyze mathematical word problem solving [11, 14]. Newman explained the five categories of student errors based on the process of solving mathematical word problem, namely: reading, comprehensive, transformation, process skills and encoding [7, 11]. Reading errors are errors the students have when reading a given problems [12-14] and simple recognition of words, symbols, and notations [7, 11]. Comprehensive errors are students' errors in understanding the meaning of a problem [7, 11]. Language interpretation errors consists of errors in reflecting everyday language into mathematical language and errors in interpreting symbols, graphs, and tables into mathematical language (Subanji and Mulyoto in [14]).

4. Conclusion

Identification of problems is an initial activity of refractive thinking. Failure to identify problems will impact the determination of problem-solving strategies. There are some errors made by the students when identifying problems in refractive thinking. Their errors in rewriting information on a given problem included in reading errors. The errors in interpreting mathematical situations rationally are language interpretation errors and comprehension errors. The students’ errors in representing ideas in the form of mathematical symbols or images are factual errors, reading errors and visualization errors in mathematical graphs. The results of this study are expected to be a reference for educators in designing learning activities so as to develop mathematical refractive thinking.

5. Acknowledgments

The authors would like to acknowledge the financial support from Institut Agama Islam Negeri Batusangkar.

6. References

[1] Maslukha M, Lukito A and Ekawati R 2018 Refractive thinking profile in solving mathematical problem reviewed from students math capability *Journal of Physics: Conference Series* **947** 012022.

[2] Prayitno A, Subanji and Muksar M 2016 Refractive thinking with dual strategy in solving mathematics problem *IOSR Journal of Research & Method in Education (IOSR-JRME)* **6** 49.

[3] Prayitno A 2016 The characteristics of students’ refractive thinking about data *Proceeding of 3rd International Conference on Research, Implementation and Education of Mathematics and Science (ICRIEMS)* (Yogyakarta) p 29.

[4] Prayitno A, Sutawidjaja A, Subanji and Muksar M 2014 Proses berpikir refraksi siswa menyelesaikan masalah data membuat keputusan *Prosiding Seminar Nasional TEQIP* (Malang: Universitas Negeri Malang) p 154.

[5] Solfitri T and Maimunah M 2018 Analysis of students errors in application integrals materials *Proceedings of the UR International Conference on Educational Sciences* (Pekanbaru: Universitas Riau) p 683.

[6] Widodo S A 2013 Analisis kesalahan dalam pemecahan masalah divergensi tipe membuktikan pada mahasiswa matematika *Jurnal Pendidikan dan Pengajaran* **46** 106.

[7] Khairani, Mukhni and Aini F Q 2019 Error analysis of college students in calculus learning *Journal of Physics: Conference Series* **1280** 042019.
[8] Siyepu S W 2015 Analysis of errors in derivatives of trigonometric functions *International Journal of STEM Education* **2** 16.
[9] Othman Z S B, Khalid A K B, and Mahat A B 2018 Students' common mistakes in basic differentiation topics *AIP Conference Proceedings* **1974** 050009.
[10] Untarti R and Kusuma A B 2019 Error identification in problem solving on multivariable calculus *Journal of Physics: Conference Series* **1188** 012064.
[11] Wijaya A, van den Heuvel-Panhuizen M, Doorman M and Robitzsch A 2014 Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors *The Mathematics Enthusiast* **11** 555.
[12] Abdullah A H, Abidin N L Z and Ali M 2015 Analysis of students' errors in solving higher order thinking skills (hots) problems for the topic of fraction *Asian Social Science* **11** 133.
[13] Rohmah M and Sutiarso S 2018 Analysis problem solving in mathematical using theory Newman *Eurasia Journal of Mathematics, Science and Technology Education* **14** 671.
[14] Baskoro I and Retnawati H 2019 Analyzing vocational school students' error in solving mathematics problems involving higher order thinking skills *Journal of Physics: Conference Series* **1320** 012102.
[15] Aditya Y and Prabawanto S 2018 Identification errors of fixed mind set student on solving mathematical deductive reasoning problem *Proceedings of International Conference on Education and Social Sciences* (Istanbul) p 779.
[16] Singh P, Rahman A A and Hoon T S 2010 The Newman procedure for analyzing primary four pupils errors on written mathematical tasks: A Malaysian perspective *Procedia Social and Behavioral Sciences* **8** 264.
[17] Usman A I 2017 Geometric error analysis in applied calculus problem solving *European Journal of Science and Mathematics Education* **5** 119.
[18] Bremigan E G 2005 An analysis of diagram modification and construction in students' solutions to applied calculus problems *Journal for Research in Mathematics Education* 248.
[19] Lai C F 2012 *Error Analysis in Mathematics* (Eugene: University of Oregon).