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Assessing Students' Pseudo-Mathematical Translation Using Translation-Verification Model

Cholis Sa'dijah[1, a)], Dona Afriyani[1, 2 b)], Subanji[1, c)], Makbul Muksar[1, d)], and Lathiful Anwar[1, e)]

[1]*Department of Mathematics Education, Universitas Negeri Malang*
Jl. Semarang 5 Malang, Indonesia, 65145
[2]*Department of Mathematics Education, Institut Agama Islam Negeri Batusangkar*
Jl. Sudirman 137 Batusangkar, Indonesia, 27127

[a)]Corresponding author: cholis.sadijah.fmipa@um.ac.id.
b)donaafriyani@iainbatusangkar.ac.id.
c)subanji.fmipa@um.ac.id.
d)makbul.muksar.fmipa@um.ac.id.
e)lathiful.anwar.fmipa@um.ac.id.

Abstract. The aims of this study is to find out the way in assesing students' pseudo-mathematical translation using translation-verification models and the characteristics of students' pseudo-mathematical translation are. This research is qualitative descriptive. Fifty-one students majoring in mathematics involved in think-aloud and interviews while solving the mathematical translation task. Data are collected and coded using rubrics containing translation activity indicators developed based on the translation-verification model. Based on the results of the assessment of the students' response using the rubric, it's obtained that the characteristics of pseudo-mathematical translation indicated by students' behavior is correctly construct the target representation of the source representation but cannot provide related verification: source and target representation attribute, conformity of connections between source and target representation attributes, representative equivalence of source and target representation. These findings can help educators recognize the weaknesses of mathematical translation and develop learning strategies to make the students' meaningful mathematical translations.

INTRODUCTION

Today, mathematics education research is more interested in seeing the fluency and flexibility of students in "doing mathematics". One of the benchmarks that can be used is the success of students in translation between different mathematical representations [1]. In addition, the researchers agreed that successful mathematical translation allowed students to have a deep mathematical understanding [2,3], developed strong mathematical connections [4–6], and succeeded in solving mathematical problems [7].

A number of studies have investigated the process of mathematical translation [4,8–10]. The results showed that the majority of students have not succeeded in performing the mathematical translation. Associated with these results, a number of researchers are also interested in seeing the types of errors that students in solving the task of mathematical translation [11–14]. Overall, previous research focuses only on students who fail in mathematical translation. In fact, students who can move from a representation to another representation may not necessarily do the real translation [13]. This condition indicates that the thinking process of students during mathematical translation is considered pseudo-thinking [15–17]. In this article, such thought processes are introduced as pseudo-mathematical translation.

The pseudo-mathematical translation is a mathematical ability that is still raw and not a real capability [17]. In other words, the completion of the written tasks of mathematical translation does not yet reflect the process of real thought. Based on a preliminary study conducted on November 7th, 2016, it was identified that the students built a target representation of the source representation correctly, but had not been able to justify the representational

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equivalence of the two representations. In this case, students completed the task of mathematical translation by imitating the previous learning experience so that the process of thinking was relatively procedural and superficial [16]. Previous research recommended teachers to recognize the weaknesses of students' mathematical translations and developed learning practices to help students to make meaningful mathematical translation [4,10,18].

This paper find out the way in assesing students' pseudo-mathematical translation using translation-verification models and the characteristics of students' pseudo-mathematical translation are. Specific research questions are how to assess pseudo-mathematical translation and how the characteristics of pseudo-mathematical translation of students are. Therefore, a rubric containing translational activity indicators is synthesized using a translation-verification model. Consideration of using this model because this model can explain students' behavior when doing mathematical translation so that it can detect students weakness during translation [13]. This research is urgent to do because the research findings can be used as an instrument to recognize pseudo-mathematical translation. In addition, the findings of this research can be used as a basis to organize the thinking process of students through the development of multiple representation based learning.

RESEARCH METHODS

This research is a qualitative descriptive study. The purposive sampling method is used to select 51 students majoring in mathematics at the Universitas Negeri Malang, Indonesia with a consideration that they have never studied calculus. Data collection was conducted in the odd semester of academic year 2017/2018, precisely from October to November 2017. Data collection was conducted in two stages: first, students were asked to think aloud when solving the mathematical translation task (MTT) and secondly, an interview was done to confirm the students thinking process during solving the MTT. The mathematical translation task can be seen in Figure 1.

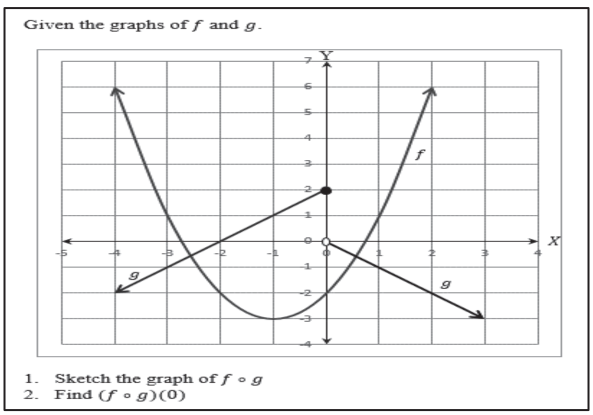

FIGURE 1. The mathematical translation task

In Figure 1 there are graphs of f and g, students are asked to sketch a composition function of f with g and determine $f \circ g(0)$. The mathematical translation task has been validated by mathematics and mathematics education experts and the result is valid to measure the students' mathematical translation. Interview's questions refer to the written response to the MTT completion and the translational-verification model activity indicators available on the assessment rubric. The assessments rubric can be seen in Table 1.

In Table 1 contains indicators of students' activity during translation based on translational-verification model. This rubric serves as a basis for identifying and formulating the characteristics of students' pseudo-mathematical translation.

TABLE 1. The Rubric of Mathematical Translation Activity Assessment Based on Translation-Verification Model

Verification	Graph to Symbols	Symbols to Graph
Attribute Construction	$A_{1.1}$ Identify the attributes of a quadratic function graph	$A_{2.1}$ Identify the symbolic attributes of a quadratic function (composition function)
	$A_{1.2}$ Identify the attributes of the linear function graph	
	$A_{1.3}$ Explain the suitability between graph attributes and symbolic attributes of a quadratic function	$A_{2.2}$ Explain the suitability between symbolic attributes and attributes of a quadratic function graph (composition function)
	$A_{1.4}$ Explain the suitability between graph attributes and symbolic attributes of the linear function	
Implementation Construction	$I_{1.1}$ Check the correctness of calculation performed during the translation from graph to symbolic	$I_{2.1}$ Check the correctness of calculation performed during translation from symbolic to graph
	$I_{1.2}$ Check the correctness of reading points at the graph	$I_{2.2}$ Check the correctness of plotting points in Cartesian coordinates
	$I_{1.3}$ Check the correctness of choosing dependent and independent variables	$I_{2.3}$ Check the correctness in sketching curves
Equivalence Construction	$E_{1.1}$ Interpret the characteristics of a quadratic function from the graph and symbolic representation	$E_{2.1}$ Interpret the characteristics of a quadratic function (composition function) from graphs and symbolic representation
	$E_{1.2}$ Interpret the characteristics of a linear function from the graph and symbolic representation	
	$E_{1.3}$ Interpret the slope of the line from symbolic and graph representation	

From the assessment results, students' written responses can be classified into two answer types i.e true and false. The false subject is not included in subsequent analysis. The written responses, think aloud, interviews and video recordings of the subjects were correctly analyzed using inductive descriptive methods [19,20]. The data is coded based on the scoring rubric so that there are several categories of mathematical translation. If the students cannot verify any of the attribute, implementation or equivalence construction, then it can be said that the students' response is classified as pseudo-mathematical translation [3,13]. Finally, the authors describe how the characteristics of students' pseudo-mathematical translation (PMT) are.

RESULT AND DISCUSSION

Students' responses during the MTT solving are grouped into five i.e. without translations (WT), true translations or real mathematical translation (RMT), real false translation (RFT), false pseudo-translation (FPT) and true pseudo-translations or pseudo-mathematical translation (PMT). The distribution of students' responses to each category is presented in Table 2.

TABLE 2. Distribution of Students' Responses During the MTT Completion

Participant	WT	RMT	RFT	FPT	PMT	Total
Frequency	2	2	26	1	20	51
Percentage	4%	4%	51%	2%	39%	100%

From Table 2, it should be noted that the highest percentage (51%) in the RFT category and the second largest (39%) in the PMT category. Specific categories of RFT have a lot of previous research documentation that describes the type of error [11–13] and the location of the error occurred [14]. That's why this research is focused on the analysis of pseudo-mathematical translation.

Referring to the written response, students included in PMT category can build target representation from source representation, either on graph translation to symbolic or vice versa. Further verification was done through interviews, obtained by the fact that PMT category students have not been able to verify attributes, implementation, or equivalence construction. The strategy used by students in solving the MTT is to translate from graph of f to its symbolic form, translate from graph of g to its symbolic form, and determine the symbolic form of the composition function f with g and the translation from symbolic of $f \circ g$ to graph.

Based on the results of the assessment on the response of PMT category, students using the rubric in Table 1, then the students can be grouped into two subcategories. The first sub-category contains 13 students, in which they cannot verify the attributes construction and implementations construction. The second sub-category contains 7 students, in which they cannot verify equivalence construction. The following descriptions of the results of the assessment of student responses based on the rubric in Table 1.

The Attribute Construction Verification

When performing translation, the first step performed by students is identifying the source representation attribute. In this step, the majority of students can identify the surface attributes of the source representation. Examples of written and spoken responses are represented by student's responses with initials SA. In the translation from graph to symbolic, SA student identifies the graph attribute of f, those are points $(0,-2),(1,1),(-2,-2),(-3.1)$ and the peak $(-1,-3)$. While for graph of g, SA student identifies two points namely (0,2) and (-2,0). This can be seen from the work of student SA in Figures 2(a) and 1(b).

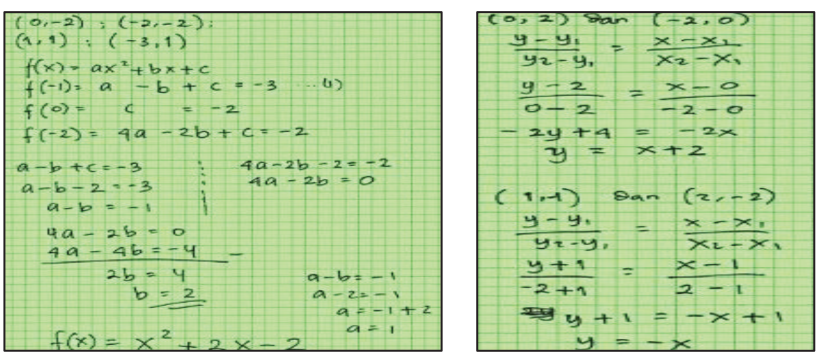

FIGURE 2. Written response of SA students in: (a) translating f graph to f symbolic, (b) translating g graph to g symbolic

At the time of the interview, SA student explained that the points were points on the graph and he had no special consideration in selecting those points. The following are the excerpts of interviews between researchers and SA student.

R: *What points are these?*
SA: *These are the points on the graphs*

R: *Do you have special consideration in selecting these points?*

SA: *No, I just select the points which can be seen clearly in the graphs.*

The same thing happened in translation from the $f \circ g$ symbolic to the $f \circ g$ graph. When performing think aloud, SA student substitutes some corresponding x values into the symbolic form of $f \circ g$, so that the ordered pairs $(x, f \circ g\ (x))$ are obtained. From the think aloud, SA student rely solely on the substitution process to find the ordered pairs. SA student does not pay attention to the symbolic representation attributes of the coefficients x^2 , the coefficients of x and the constants [9]. SA students can only recognize the explicit attributes of a quadratic function graphs and linear function graphs [21] . This finding is consistent with previous research [22] that students find it difficult to identify attributes of graph and symbolic representation because these two representations have high attribute densities. Referring to the assessment rubric, SA student cannot verify the indicators $A_{1.1}$, $A_{1,2}$, and $A_{2.1}$ in the attribute construction. From the assessment results obtained the first characteristic of students' pseudo-mathematical translation is correct in solving the MTT, but cannot verify the attributes of source and target representation.

After identifying the source representation attribute, the second step in translation is performing initial coordination by mapping the source representation attribute to the target representation attribute. For example, on the graph f translation to symbolic f, student SA maps a parabolic graph to a quadratic function in the form $f\ (x) = ax^2 + bx + c$ and substituting the coordinates $(-1,-3), (0,-2)$ and $(-2,-2)$ to $f\ (x) = ax^2 + bx + c$. The coordination performed by SA student can be seen in Figure 2(a). In the translation from graph of g to symbolic, SA student maps the line-shaped graph to the general form of the line equation as shown in Figure 2(b). The coordination by SA student is procedural and the mapping is fact mapping [22]. That is, SA student cannot see the compatibility of connections between f graph's attributes and f symbolic attributes [4]. This is because SA student does not recognize the attribute of the graph representation as a source representation [13].

In the translation from symbolic of $f \circ g$ to graph, SA student maps the ordered pair $(x, f \circ g(x))$ to the coordinates in the $f \circ g$ graph. Based on the results of the interview, SA student revealed that he chose randomly the value of x to be substituted into the symbolic of $f \circ g$. Here is the excerpt of the interview.

R : *How did you draw the $f \circ g$ graph* ?
SA: *Choose the coordinates randomly in the domain $f \circ g$ then substitute it to $f \circ g$. So that I get the coordinates to be plotted in the cartesius coordinates.*

SA student does not make connections between coefficients of x^2 and the direction of the parabolic opening, the connection between the constants and the y value at the intersection point of the curve and the x-axis. Such connections are called isomorphic connections [23]. Referring to the assessment rubric, SA student's response during coordination is not based on the compatibility of the connections between the source and target representation attributes. In other words, SA student cannot verify indicators $A_{1.3}$, $A_{1.4}$ and $A_{2.2}$ in attribute construction. From the assessment results obtained the characteristics of both pseudo-mathematical translation is true in solving the MTT, but cannot verify the compatibility of connections between attributes of source and target representation.

The Implementation Verification

All students who are in the PMT category can perform activities on implementation construction. This finding is in accordance with the findings of previous research [13] that is the students who can build targets correctly can avoid algorithm errors. Although all PMT category students can verify the construction of the implementation, but for a sample of the students' response is represented only by the student with the initials SB. In SB student's work, it appears that he can read the points on the graph and write in the function. SB student's work as presented in Figure 3.

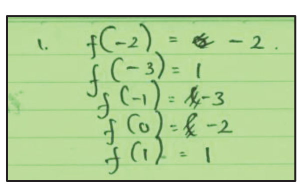

FIGURE 3. The result of SB student in reading points in the f graph

In the interview, SB student can show the point on the graph. Here is the excerpt from the interview.

R : *Please, show me a point in the f graph* ?
SB : $f(-2) = (-2)$ *are the points* $(-2, -2)$. *Here is the points (showing the points* $(-2, -2)$ *in f graph).*

From Figure 3 it is also seen that SB student can determine which are the dependent and independent variables of the function. SB student selects the value on the horizontal axis as the value of the independent variable and the value on the vertical axis as the value of the dependent variable.

When constructing the graph from the symbolic $f \circ g$, SB student is correct in plotting the points on the cartesian coordinates and connecting the points to form a smooth curve. This activity can be seen in Figure 4.

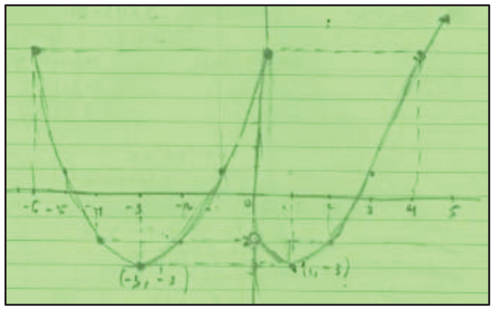

FIGURE 4. The result of SB student's work in constructing a $f \circ g$ graph

Referring to the scoring rubric, it is found that SB student can verify indicators $I_{1.2}$, $I_{1.3}$, $I_{2.2}$ and $I_{2.3}$ on the attributes construction.

Whether on translation from graph to symbolic or the vice versa, SB student checks the correctness of the resulting target representation. In translation from graph to symbolic, SB student substitutes two points on the graph of f to symbolic so that the point satisfies the equation. For more details it can be seen in Figure 5.

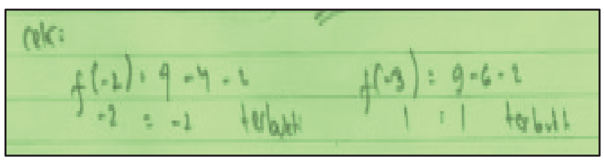

FIGURE 5. The result of SB student's work in checking the correctness of $f \circ g$ graph.

On translation from symbolic to graph, SB student can determine the value of $f \circ g$ (0) from the symbolic form of $f \circ g$ as shown in Figure 6. However, SB student cannot yet show the $f \circ g$ (0) value of the graph. This can be seen from the following interview excerpt.

R : How can you get $f \circ g(0)$?
SB: I substitute $x = 0$ to $f \circ g(x)$ so I get $f \circ g(0) = 6$.
R : Is there any other way?
SB: No

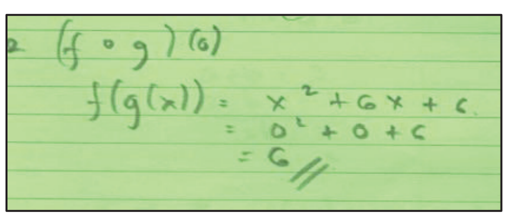

FIGURE 6. The result of SB student's work in determining $f \circ g(0)$ from symbolic form

Referring to the assessment rubric, it is found that SB student can verify the indicator $I_{1.1}$, but not yet complete for indicator $I_{2.1}$ in the implementation construction.

The Equivalency Verification

The final step in translation is to determine the representational equivalence of the source and target representation. Students' response to equivalent construction verification is represented by student's response with initials SB. From the task-based interviews, it was found that SB student could explain the characteristics of a quadratic function from symbolic and graph representation. Student SB understands that the quadratic function in graph form can be seen from the curve-like shape of a parabola and has a peak point, whereas in symbolic form is marked with the highest rank of variable X that is two. Likewise, for a linear functions, SB students understand that a linear function in the form of graphs can be seen from the line-shaped graph. While the symbolic form is marked with the highest rank of variable X that is one.

SB student can give an interpretation of characteristic of a quadratic and linear function from the graph and symbolic representation. However, the interpretation is local [22] and at the level of external identification [24]. SB student has not been able to provide a global interpretation of the qualitative relationship between the horizontal and vertical axes on the graph or association between the dependent and independent variables on the symbolic form [9]. Referring to the assessment rubric, it can be said that SB student has not been able to verify indicators $E_{1.1}$ and $E_{1.2}$ in equivalence construction.

SB students can only interpret the concept of composition function f with g of symbolic representation, but not yet from the graph. During the interview, SB student explained that symbolic g is the domain of the symbolic $f \circ g$. In addition, SB student has not been able to interpret gradients from graphs of g. At the time of the interview, SB student was only able to explain that the gradient could be seen from the coefficient of x on the symbolic form of function. Here are excerpts of interviews between SB student and researchers.

R : *Could you please explain the quadratic function in the form of symbolic and graph* ?
SB: *Graph is in the form of a curve with a vertex, it is a graph for a quadratic function, on the other hand, the symbolic form can be seen the existence Because the highest degree of x is two.*
R : *Could you please explain the a linear function in the form of symbolic and graph* ?
SB: The *Linear function graph is in the form of a straight line however in the symbolic form, the degree of x is one.*
R : *Please explain the concept of composition function of f and g from the graph and symbolic form*?
SB: *Here we have symbolic f and g, so the composition function of f and g is g function worked for f. Or g function is the domain of $f \circ g$. From the graph, I am a little bit confused to explain because f and g functions cannot be seen in the $f \circ g$ graph. In the $f \circ g$ graph, there is only a quadratic function.*
R *: Would you please explain the gradient of g from graph and symbolic form* ?
SB: *The gradient of $g(x) = x + 2$ is one. I get it from the coefficient of x. The gradient of graph. Hmmm. What is it* ?

SB student is not familiar with the presentation of compositional functions in graph form and is not used to explaining gradient of graphs. This shows that students' understanding of the concept of compositional function and the gradient is limited to the symbolic form [6]. In addition, SB student does not yet have a deep understanding of composition and gradient of functions [25]. One reason is that SB student only follows procedures based on prior experience without knowing the reason why the procedure is used [26]. Based on the result of the students' response on equivalence construction, the third characteristic of pseudo-mathematical translation of the student is correct in solving the MTT but cannot yet verify the corresponding representational of the source and target representation.

CONCLUSION

A Finally, this study resulted in pseudo-mathematical translation assessment method using rubric developed based on translational-verification model. The assessment rubric contains activity indicators to be performed during translation that includes attribute construction, implementation, and equivalence. The criteria used for the assessment is the inability of the students to verify the indicator on the rubric. From the results of this assessment, it was obtained the mathematical characteristic of students' pseudo-mathematical translation that is building targets correctly but cannot verify: attributes of source and target representation, matching connections between attributes of both representations, and representational equivalence of both representations. This research leaves the question of whether the three characteristics of pseudo-mathematical translation are tiered. Therefore, the next research is suggested to develop a scale of pseudo-mathematical translation assessment, so that it can measure the pseudo-mathematical translation level. Furthermore, this research is still limited to true pseudo-translation, so it is advisable to investigate how to measure the false pseudo-translation.

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